Dyadic Green's Function of Multilayer Cylindrical Closed and Sector Structures for Waveguide, Microstrip-Antenna, and Network Analysis

Michael Thiel and Achim Dreher, *Senior Member, IEEE*

***Abstract*—A clear and systematic method to derive the spectral- and space-domain dyadic Green's function of arbitrary cylindrical multilayer and multiconductor structures is proposed. The derivation is either done for a circumferentially closed or a cylindrical sector structure, which is bounded by electric or magnetic walls in an azimuthal direction. The solution for the dyadic Green's function in the spectral domain is obtained via an equivalent circuit. Relations between the spectral and space domains for the dyadic Green's functions are derived using eigensolution expansions. Finally, the dyadic Green's function is applied to the problem of finding the propagation constants of the two-layer dielectric rod.**

***Index Terms*—Cylindrical microstrip networks and antennas, cylindrical sector structures, cylindrical waveguides, dyadic Green's function.**

I. Introduction

FOR FUTURE RF applications, more and more conformal antennas are needed. In many cases, cylindrical microstrip antennas fulfill the desired properties. If the analysis is done with an integral-equation technique, the dyadic Green's function may serve as kernel of the formulation. The Green's function can also be used for the analysis of waveguiding structures, which do not contain any metallizations.

Solutions for dyadic Green's functions of closed cylindrical structures in the spectral domain can be found in [1] and [2], for space domain in [3] and [4], and for perfectly conducting cylindrical sector structures in [5].

This paper presents the derivation of the spectral- and space-domain dyadic Green's function for closed cylindrical and cylindrical sector structures consisting of an arbitrary number of layers with metallizations in the interfaces. To obtain the spectral-domain solution, a known equivalent-circuit approach [6] has been applied to the cylindrical problem. Important relations for the transformation of the spectral-domain solution in the space domain have been derived by means of an eigensolution expansion.

The presented way to determine the Green's dyad can be applied to problems of microstrip or network analysis, e.g., in combination with the spectral-domain approach. To demonstrate the applicability to waveguiding structures, an example is shown below.

Manuscript received July 30, 2001.

The authors are with the DLR Institute of Communications and Navigation, German Aerospace Center, D-82234 Wessling, Germany.

Digital Object Identifier 10.1109/TMTT.2002.804637

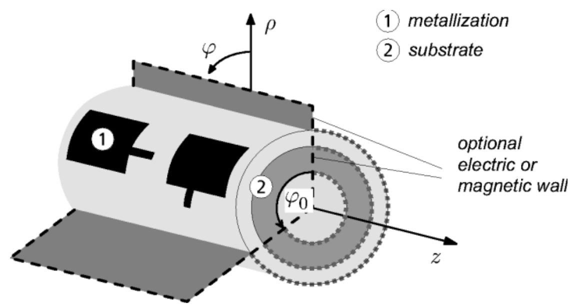


Fig. 1. Multilayer cylindrical microstrip structure and coordinate definitions.

II. Analysis

The structure may appear as is shown in Fig. 1, along with the corresponding coordinate definitions. In the z-direction, the extension of the structure is infinite. The layers are assumed to be homogeneous and in an azimuthal direction either bounded by electric or magnetic walls or unbounded so that the circumference of the structure is closed. In such a structure, the solution of waveguide or microstrip structures can be reduced to system equations, which involve only the tangential field components in the interfaces of the layered structure. The dyadic Green's function fulfilling the inhomogeneous vector-wave equation

$$\left(\nabla\times\nabla\times -k^2\right)\boldsymbol{G}\left(\mathbf{r}-\mathbf{r}'\right) = -j\omega\mu\boldsymbol{I}\delta\left(\mathbf{r}-\mathbf{r}'\right) \tag{1}$$

where $\boldsymbol{I}$ is the unitary dyad can be written as

$$\boldsymbol{G}_{kk'}\left(\varphi-\varphi', z-z'\right) = \begin{bmatrix} G_{\varphi\varphi,kk'} & G_{\varphi z,kk'} \\ G_{z\varphi,kk'} & G_{zz,kk'} \end{bmatrix}. \tag{2}$$

$\boldsymbol{G}_{kk'}$ is used to set up a relation between the electric current in the interface k' and the electric field in the interface k.

The procedure to derive the Green's dyad starts from the source-free wave equation in cylindrical coordinates

$$\left(\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial\varphi^2}+\rho^2\left(\varepsilon_r\mu_r+\frac{\partial^2}{\partial z^2}\right)\right)\psi = 0 \tag{3}$$

for the independent tangential-field components $\psi = E_z, H_z$. A time-dependence $\exp(j\omega t)$ has been assumed and the longitudinal and radial coordinates, as well as the spectral propagation

TABLE I
EIGENSOLUTIONS ACCORDING TO BOUNDARY COMBINATIONS [ELECTRIC (el.), MAGNETIC (mag.), PERIODIC (per.)]

Boundary combinations	$\psi = E_z$ Mode i	$\psi = H_z$ Mode i	ν_i
el./el.	$\sin(\nu_i\varphi)$	$\cos(\nu_i\varphi)$	$\frac{i\pi}{\varphi_0}$
mag./el.	$\cos(\nu_i\varphi)$	$\sin(\nu_i\varphi)$	$\frac{(i-\frac{1}{2})\pi}{\varphi_0}$
periodic	$\sin(\nu_i\varphi+\phi)$	$\cos(\nu_i\varphi+\phi)$	i
per. with (7)	$\exp(-j\nu_i\varphi)$	$\exp(-j\nu_i\varphi)$	i

constants, have been normalized by k_0.[1] The remaining tangential field components are given through

$$\left(\varepsilon_r\mu_r + \frac{\partial^2}{\partial z^2}\right)\begin{bmatrix} E_\varphi \\ \eta_0 H_\varphi \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho}\frac{\partial^2}{\partial z \partial\varphi} & j\mu_r \frac{\partial}{\partial\rho} \\ -j\varepsilon_r \frac{\partial}{\partial\rho} & \frac{1}{\rho}\frac{\partial^2}{\partial z\partial\varphi} \end{bmatrix}\begin{bmatrix} E_z \\ \eta_0 H_z \end{bmatrix} \tag{4}$$

with $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$.

As a general solution of (3), we use the eigensolution or modal expansions of the field components in the φ-direction given by one of the following Fourier series:[2]

$$\psi(\rho,\varphi) = -j\frac{2}{\varphi_0}\sum_{i=0}^{\infty}\widetilde{\psi}^{(i)}(\rho)\sin(\nu_i\varphi) = \mathbf{t}_s\cdot\widetilde{\mathbf{\Psi}} \tag{5}$$

$$\psi(\rho,\varphi) = \frac{1}{\varphi_0}\left(\widetilde{\psi}^{(0)} + 2\sum_{i=1}^{\infty}\widetilde{\psi}^{(i)}(\rho)\cos(\nu_i\varphi)\right) = \mathbf{t}_c\cdot\widetilde{\mathbf{\Psi}} \tag{6}$$

$$\psi(\rho,\varphi) = \frac{1}{\varphi_0}\sum_{i=-\infty}^{\infty}\widetilde{\psi}^{(i)}(\rho)\exp(-j\nu_i\varphi) = \mathbf{t}_e\cdot\widetilde{\mathbf{\Psi}} \tag{7}$$

with $\psi = E_z, H_z$ and the boundary combinations in Table I. $\psi = \mathbf{t}\cdot\widetilde{\mathbf{\Psi}}$ is interpreted as scalar product and $\widetilde{\mathbf{\Psi}}$ only contains the coefficients $\widetilde{\psi}^{(i)}$. The angle $\varphi_0 = \varphi_o - \varphi_u$ is defined in Fig. 1, where $\varphi_u = 0$, $\varphi_o = \varphi_0$, and $0 < \varphi_0 \leq 2\pi$. If there are no walls and the circumference of the structure is closed, periodic solutions are obtained. For that, the complex Fourier series (7) can be used and additionally assuming a wave propagation $e^{-jk_z z}$ in z-direction, which implies a complex Fourier transform along the z-coordinate, the two-dimensional Fourier transform

$$\widetilde{\psi}^{(i)}(k_z) = \int_0^{2\pi}\int_{-\infty}^{\infty}\psi(\varphi,z)e^{jk_z z}e^{j\nu_i\varphi}dz d\varphi \tag{8}$$

$$\psi(\varphi,z) = \frac{1}{4\pi^2}\sum_{i=-\infty}^{\infty}\int_{-\infty}^{\infty}\widetilde{\psi}^{(i)}(k_z)e^{-jk_z z}e^{-j\nu_i\varphi}dk_z \tag{9}$$

is obtained for a closed cylindrical structure without azimuthal boundaries. Using (8), the wave equation (3) transforms into the

[1] $\rho = \bar{\rho}k_0$, $z = \bar{z}k_0$, $k_\rho = \bar{k}_\rho/k_0$, and $k_z = \bar{k}_z/k_0$, where the unnormalized quantities are indicated by a bar

[2] For reasons of simplicity, the sine-series starts with $i = 0$.

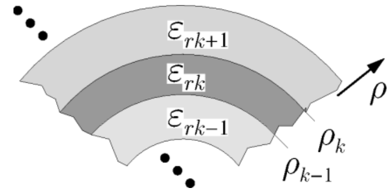


Fig. 2. Stratified dielectric with corresponding notations.

spectral domain

$$\left(\boldsymbol{I}\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) - \boldsymbol{\nu}^2 + (k_\rho\rho)^2\boldsymbol{I}\right)\widetilde{\mathbf{\Psi}} = 0 \tag{10}$$

with $k_\rho^2 = \varepsilon_r\mu_r - k_z^2$ and the diagonal matrix $\boldsymbol{\nu} = \mathrm{diag}(\nu_i)$ following from the vectorial notation in (5)–(7). $\boldsymbol{I}$ is the unitary dyad. For cylindrical sector structures, the fields in wave equation (3) are replaced by the modal expansions in (5) and (6) according to Table I. Combining the orthogonality properties of the expansion functions and a complex Fourier transform along z, the wave equation in the spectral domain (10) is also obtained.

Within an arbitrary layer k (Fig. 2), the general solutions of these Bessel differential equations (10) are composed of Bessel and Neumann functions

$$\widetilde{\mathbf{\Psi}}_k = \boldsymbol{J}_{\boldsymbol{\nu}}(k_{\rho k}\rho)\mathbf{A}_k + \boldsymbol{Y}_{\boldsymbol{\nu}}(k_{\rho k}\rho)\mathbf{B}_k, \qquad \rho_{k-1}\leq\rho\leq\rho_k \tag{11}$$

where the matrix notation $\boldsymbol{\mathcal{C}}_{\boldsymbol{\nu}} = \mathrm{diag}(\mathcal{C}_{\nu_i})$ is used. If the inner layer extends to $\rho = 0$, the fields must be finite in this point so that

$$\widetilde{\mathbf{\Psi}}_0 = \boldsymbol{J}_{\boldsymbol{\nu}}(k_{\rho 0}\rho)\mathbf{A}_0, \qquad 0\leq\rho\leq\rho_0 \tag{12}$$

has to be chosen, and if the structure is radially open, the radiation condition must be fulfilled necessitating

$$\widetilde{\mathbf{\Psi}} = \boldsymbol{H}_{\boldsymbol{\nu}}^{(2)}(k_{\rho n}\rho)\mathbf{A}_n, \qquad \rho_{n-1}\leq\rho \tag{13}$$

within the outer layer ($\rho \rightarrow \infty$). The sign of $k_\rho = \pm\sqrt{\varepsilon_r\mu_r - k_z^2}$ has to be chosen such that

$$\mathrm{Re}(k_\rho)\geq 0 \text{ or } \mathrm{Im}(k_\rho)\leq 0 \tag{14}$$

holds to get only outgoing or decaying waves according to the Sommerfeld's radiation condition [7]. By using (11) and the derivative with respect to ρ on both sides of the layer k, the unknown coefficient vectors $\mathbf{A}_k$ and $\mathbf{B}_k$ are eliminated and the following relation is set up:

$$\frac{d}{d\rho}\begin{bmatrix}\widetilde{\mathbf{\Psi}}_{k-1}\\ \widetilde{\mathbf{\Psi}}_k\end{bmatrix} = \hat{\boldsymbol{p}}_{\boldsymbol{\nu}k}\underbrace{\begin{bmatrix}\bar{\boldsymbol{r}}_{\boldsymbol{\nu}k} & \frac{2}{\pi}\boldsymbol{I}\\ -\frac{2}{\pi}\boldsymbol{I} & \bar{\boldsymbol{q}}_{\boldsymbol{\nu}k}\end{bmatrix}}_{\mathbf{\Gamma}_k}\begin{bmatrix}\widetilde{\mathbf{\Psi}}_{k-1}\\ \widetilde{\mathbf{\Psi}}_k\end{bmatrix}. \tag{15}$$

Herein are

$$\begin{aligned}\hat{\boldsymbol{p}}_{\boldsymbol{\nu}k} &= \mathrm{diag}\left(\boldsymbol{p}_{\boldsymbol{\nu}k}\rho_{k-1}^{-1}, \boldsymbol{p}_{\boldsymbol{\nu}k}\rho_k^{-1}\right)\\ \bar{\boldsymbol{q}}_{\boldsymbol{\nu}k} &= k_{\rho k}\rho_k\boldsymbol{q}_{\boldsymbol{\nu}k}\\ \bar{\boldsymbol{r}}_{\boldsymbol{\nu}k} &= k_{\rho k}\rho_{k-1}\boldsymbol{r}_{\boldsymbol{\nu}k}\\ \bar{\boldsymbol{s}}_{\boldsymbol{\nu}k} &= k_{\rho k}^2\rho_k\rho_{k-1}\boldsymbol{s}_{\boldsymbol{\nu}k}\end{aligned} \tag{16}$$

with the cross-products of Bessel functions [8] $\boldsymbol{p}_{\boldsymbol{\nu}k}$, $\boldsymbol{q}_{\boldsymbol{\nu}k}$, $\boldsymbol{r}_{\boldsymbol{\nu}k}$, and $\boldsymbol{s}_{\boldsymbol{\nu}k}$ and the matrix notation $\boldsymbol{\mathcal{C}}_{\boldsymbol{\nu}k} = \boldsymbol{\mathcal{C}}_{\boldsymbol{\nu}}(k_{\rho k}\rho_k)$ and $\boldsymbol{\mathcal{C}}_{\boldsymbol{\nu}k-1} = \boldsymbol{\mathcal{C}}_{\boldsymbol{\nu}}(k_{\rho k}\rho_{k-1})$. Applying (8) to (4) or using the modal expansions (5) and (6) in combination with the Fourier transform along the z-coordinate, (4) is given in the spectral domain by

$$\left(\varepsilon_r\mu_r - k_z^2\right)\begin{bmatrix}\widetilde{\mathbf{E}}_\varphi\\ \eta_0\widetilde{\mathbf{H}}_\varphi\end{bmatrix} = \begin{bmatrix}-\frac{1}{\rho}k_z\boldsymbol{\nu} & j\mu_r\boldsymbol{I}\frac{\partial}{\partial\rho}\\ -j\varepsilon_r\boldsymbol{I}\frac{\partial}{\partial\rho} & -\frac{1}{\rho}k_z\boldsymbol{\nu}\end{bmatrix}\begin{bmatrix}\widetilde{\mathbf{E}}_z\\ \eta_0\widetilde{\mathbf{H}}_z\end{bmatrix}. \tag{17}$$

The derivation with respect to ρ is replaced by (15) and, after some algebraic manipulations, one finally obtains the hybrid matrix form ([6, eq. (18)]) with the matrix $\widetilde{\boldsymbol{K}}_k$ where the definitions

$$\widetilde{\mathbf{E}}_k = j\begin{bmatrix}\rho_k\widetilde{\mathbf{E}}_{\varphi k}\\ \widetilde{\mathbf{E}}_{zk}\end{bmatrix} \text{ and } \widetilde{\mathbf{H}}_k = -\eta_0\begin{bmatrix}-\widetilde{\mathbf{H}}_{zk}\\ \rho_k\widetilde{\mathbf{H}}_{\varphi k}\end{bmatrix} \tag{18}$$

are used in the cylindrical case. The submatrices of $\widetilde{\boldsymbol{K}}_k$ constitute the following:

$$\begin{aligned}\widetilde{\boldsymbol{V}}_k &= \frac{\pi}{2}\begin{bmatrix}\bar{\boldsymbol{q}}_{\boldsymbol{\nu}} & k_z k_\rho^{-2}\boldsymbol{\nu}\left(\bar{\boldsymbol{r}}_{\boldsymbol{\nu}}+\bar{\boldsymbol{q}}_{\boldsymbol{\nu}}\right)\\ \mathbf{0} & -\bar{\boldsymbol{r}}_{\boldsymbol{\nu}}\end{bmatrix}_k\\ \widetilde{\boldsymbol{Z}}_k &= \frac{\pi}{2\varepsilon_r}\begin{bmatrix}-\left(\varepsilon_r\mu_r\bar{\boldsymbol{s}}_{\boldsymbol{\nu}}+k_z^2\boldsymbol{\nu}^2\boldsymbol{p}_{\boldsymbol{\nu}}\right)k_\rho^{-2} & k_z\boldsymbol{\nu}\boldsymbol{p}_{\boldsymbol{\nu}}\\ k_z\boldsymbol{\nu}\boldsymbol{p}_{\boldsymbol{\nu}} & -k_\rho^2\boldsymbol{p}_{\boldsymbol{\nu}}\end{bmatrix}_k\\ \widetilde{\boldsymbol{Y}}_k &= \frac{\pi}{2\mu_r}\begin{bmatrix}k_\rho^2\boldsymbol{p}_{\boldsymbol{\nu}} & k_z\boldsymbol{\nu}\boldsymbol{p}_{\boldsymbol{\nu}}\\ k_z\boldsymbol{\nu}\boldsymbol{p}_{\boldsymbol{\nu}} & \left(\varepsilon_r\mu_r\bar{\boldsymbol{s}}_{\boldsymbol{\nu}}+k_z^2\boldsymbol{\nu}^2\boldsymbol{p}_{\boldsymbol{\nu}}\right)k_\rho^{-2}\end{bmatrix}_k\\ \widetilde{\boldsymbol{B}}_k &= \frac{\pi}{2}\begin{bmatrix}-\bar{\boldsymbol{r}}_{\boldsymbol{\nu}} & \mathbf{0}\\ -k_z k_\rho^{-2}\boldsymbol{\nu}\left(\bar{\boldsymbol{r}}_{\boldsymbol{\nu}}+\bar{\boldsymbol{q}}_{\boldsymbol{\nu}}\right) & \bar{\boldsymbol{q}}_{\boldsymbol{\nu}}\end{bmatrix}_k.\end{aligned} \tag{19}$$

To obtain the admittance matrix ([6, eq. (23)]) in cylindrical coordinates, we combine (12) and (13) with (17) so that

$$\widetilde{\boldsymbol{Y}}_{0,n} = \frac{\mp\boldsymbol{u}_{\boldsymbol{\nu}0,n}^{-1}}{\mu_{0,n}k_{\rho0,n}\rho_{0,n-1}}\begin{bmatrix}-k_{\rho0,n}^2 & -k_z\boldsymbol{\nu}\\ -k_z\boldsymbol{\nu} & -\boldsymbol{y}_{0,n}\end{bmatrix} \tag{20}$$

with

$$\boldsymbol{y}_{0,n} = \frac{k_z^2}{k_{\rho0,n}^2}\boldsymbol{\nu}^2 - \varepsilon_{r0,n}\mu_{r0,n}\rho_{0,n-1}^2\boldsymbol{u}_{\boldsymbol{\nu}0,n}^2 \tag{21}$$

and

$$\boldsymbol{u}_{\boldsymbol{\nu}0} = \boldsymbol{J}'_{\boldsymbol{\nu}}\left(k_{\rho0}\rho_0\right)\boldsymbol{J}_{\boldsymbol{\nu}}^{-1}\left(k_{\rho0}\rho_0\right) \tag{22}$$

$$\boldsymbol{u}_{\boldsymbol{\nu}n} = \boldsymbol{H}'^{(2)}_{\boldsymbol{\nu}}\left(k_{\rho n}\rho_{n-1}\right)\boldsymbol{H}_{\boldsymbol{\nu}}^{(2)^{-1}}\left(k_{\rho n}\rho_{n-1}\right). \tag{23}$$

Continuity is taken into account ([6, eq. (25)]) with the cylindrical definition of the current $\widetilde{\mathbf{J}}_m = -\eta_0[\,\widetilde{\mathbf{J}}_{\varphi m} \quad \rho_k\widetilde{\mathbf{J}}_{zm}\,]^t$.

The dyadic Green's function in the spectral domain is now derived [6] so that

$$\widetilde{\mathbf{E}} = \widetilde{\boldsymbol{G}}\cdot\widetilde{\mathbf{J}} \tag{24}$$

is obtained. The solution in the spectral domain is applicable for closed cylindrical and cylindrical sector structures.

The Green's function is defined as the response of the electric field to a unit source [9]. The electric field from (24) is transformed to the space domain by (9) and, for the current, a point source with amplitude 1 is used. The relation between the spectral and space domains for the dyadic Green's function of closed cylindrical structures is then described by

$$\begin{aligned}&\boldsymbol{G}_{kk'}\left(\varphi-\varphi', z-z'\right)\\ &= \frac{1}{4\pi^2\cdot\rho_{k'}}\sum_{i=-\infty}^{\infty}\int_{-\infty}^{\infty}\widetilde{\boldsymbol{G}}_{kk'}^{(i)}\left(k_z\right)\cdot e^{-jk_z(z-z')}e^{-j\nu_i(\varphi-\varphi')}dk_z\end{aligned} \tag{25}$$

where $\widetilde{\boldsymbol{G}}_{kk'}^{(i)}$ belongs to a single mode in the spectral domain.

For cylindrical-sector structures, the modal expansions (5) and (6) have to be taken into account for the fields and sources. Together with the Fourier transform along z, the space-domain solution in case of electric walls is obtained [10] as follows:

$$\begin{aligned}&\boldsymbol{G}_{\varphi\varphi,kk'}\left(\varphi-\varphi', z-z'\right)\\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{\varphi_0\rho_{k'}}\widetilde{\mathbf{G}}_{\varphi\varphi,kk'}^{(0)}\left(k_z\right) + \frac{2}{\varphi_0\rho_{k'}}\sum_{i=1}^{\infty}\widetilde{\mathbf{G}}_{\varphi\varphi,kk'}^{(i)}\left(k_z\right)\right.\\ &\qquad\left.\cdot\cos\left(\nu_i\varphi'\right)\cos\left(\nu_i\varphi\right)\right)e^{-jk_z(z-z')}dk_z\end{aligned} \tag{26}$$

$$\begin{aligned}&\boldsymbol{G}_{\varphi z,kk'}\left(\varphi-\varphi', z-z'\right)\\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2j}{\varphi_0\rho_{k'}}\sum_{i=1}^{\infty}\widetilde{\mathbf{G}}_{\varphi z,kk'}^{(i)}\left(k_z\right)\cdot\sin\left(\nu_i\varphi'\right)\cos\left(\nu_i\varphi\right)\\ &\quad\cdot e^{-jk_z(z-z')}dk_z\end{aligned} \tag{27}$$

$$\begin{aligned}&\boldsymbol{G}_{z\varphi,kk'}\left(\varphi-\varphi', z-z'\right)\\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(-2j)}{\varphi_0\rho_{k'}}\sum_{i=1}^{\infty}\widetilde{\mathbf{G}}_{z\varphi,kk'}^{(i)}\left(k_z\right)\cdot\cos\left(\nu_i\varphi'\right)\sin\left(\nu_i\varphi\right)\\ &\quad\cdot e^{-jk_z(z-z')}dk_z\end{aligned} \tag{28}$$

$$\begin{aligned}&\boldsymbol{G}_{zz,kk'}\left(\varphi-\varphi', z-z'\right)\\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{\varphi_0\rho_{k'}}\sum_{i=1}^{\infty}\widetilde{\mathbf{G}}_{zz,kk'}^{(i)}\left(k_z\right)\cdot\sin\left(\nu_i\varphi'\right)\sin\left(\nu_i\varphi\right)\\ &\quad\cdot e^{-jk_z(z-z')}dk_z.\end{aligned} \tag{29}$$

The relations for a combination of magnetic/magnetic or electric/magnetic walls are obtained in the same way.

III. Application

As an example, we consider the twice-stratified dielectric rod in Fig. 3. The following system equation:

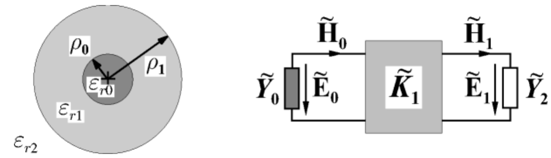
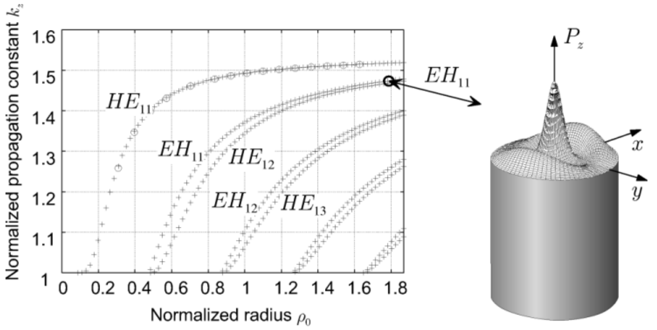


Fig. 3. Twice-stratified dielectric rod.

$$\widetilde{\boldsymbol{Y}}\widetilde{\mathbf{E}}_1 = \left(\widetilde{\boldsymbol{Y}}^{(u)} + \widetilde{\boldsymbol{Y}}_2\right)\widetilde{\mathbf{E}}_1 = 0 \quad\Leftrightarrow\quad \det\left(\widetilde{\boldsymbol{Y}}\left(k_z\right)\right) = 0 \tag{30}$$

with

$$\widetilde{\boldsymbol{Y}}^{(u)} = \left(\widetilde{\boldsymbol{Y}}_1 + \widetilde{\boldsymbol{B}}_1\widetilde{\boldsymbol{Y}}_0\right)\left(\widetilde{\boldsymbol{V}}_1 + \widetilde{\boldsymbol{Z}}_1\widetilde{\boldsymbol{Y}}_0\right)^{-1} \tag{31}$$



(a) (b)

Fig. 4. (a) Propagation constants of the dielectric rod in Fig. 3, normalized by k_0. (b) Intensity of energy flow, represented by Poynting vector P_z. $f_0 = 10^{13}$ Hz, $\bar{\rho}_0 = 8.55 \cdot 10^{-6}$ m, $\rho_0 = \bar{\rho}_0 k_0$, $\rho_1 = 7 \cdot \rho_0$, $\varepsilon_{r0} = 2.56$, $\varepsilon_{r1} = 2.3104$, and $\varepsilon_{r2} = 1$.

has to be solved to obtain the propagation constants k_z. We compared the dominant mode to the result obtained with Ansoft HFSS (○) [see Fig. 4(a)], where higher order modes could not be identified directly. The intensity of energy flow [see Fig. 4(b)] is given by the Poynting vector

$$\mathbf{P} = \frac{1}{2}\Re\left(\mathbf{E} \times \mathbf{H}^*\right) \cdot \mathbf{e}_z \tag{32}$$

in the z-direction, where $\mathbf{E} = [E_\rho, E_\varphi]$ and $\mathbf{H} = [H_\rho, H_\varphi]$. It has been computed for the $\mathbf{EH}_{11}$ mode marked in Fig. 4(a) with the normalized propagation constant $k_z = 1.47429$.

IV. Conclusion

A systematic approach to derive the dyadic Green's function in the spectral domain has been applied to cylindrical coordinates. The procedure is valid for closed cylindrical and cylindrical sector structures. For both, necessary relations between the spectral and space domains have been derived. With these, the reaction integrals in the method of moments (MoM) formalism can be transformed into the spectral domain. This enables the analysis of microstrip antennas and networks on multilayer cylindrical closed and sector structures with the spectral-domain approach using the presented dyadic Green's functions. Through the solution of a system equation, the propagation constants of a waveguide structure were obtained.

References

[1] A. Nakatani and N. G. Alexopoulos, "Microstrip elements on cylindrical substrates—General algorithm and numerical results," *Electromagnetics*, no. 9, pp. 405–426, 1989.

[2] K.-L. Wong, *Design of Nonplanar Microstrip Antennas and Transmission Lines*. New York: Wiley, 1999.

[3] F. C. Silva *et al.*, "Analysis of microstrip antennas on circular cylindrical substrates with a dielectric overlay," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1398–1404, Sept. 1991.

[4] Z. Xiang and Y. Lu, "Electromagnetic dyadic Green's function in cylindrical multilayered media," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 614–621, Apr. 1996.

[5] J. R. Wait, *Electromagnetic Radiation From Cylindrical Structures*. New York: Pergamon, 1959.

[6] A. Dreher, "A new approach to dyadic Green's function in spectral domain," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1297–1302, Nov. 1995.

[7] A. Sommerfeld, *Partial Differential Equations in Physics*. New York: Academic, 1967.

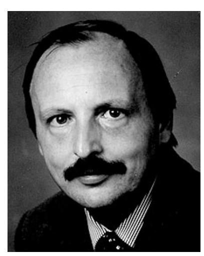
[8] M. Abramovitz and A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1964.

[9] R. E. Collin, *Field Theory of Guided Waves*. New York: IEEE Press, 1991.

[10] M. Thiel and A. Dreher, "Eigensolution expansion of dyadic Green's function for the analysis of microstrip antennas on cylindrical sector multilayer structures," in *IEEE AP-S Int. Symp. Dig.*, Boston, MA, July 2001, pp. 272–275.

Michael Thiel was born in Esslingen a.N., Germany, in 1971. He received the Dipl.-Ing. (M.S.) degree in electrical engineering from the University of Stuttgart, Stuttgart, Germany, in 1996, and is currently working toward the Dr.-Ing. (Ph.D.) degree at the Institute of Communications and Navigation, Oberpfaffenhofen, Germany, in cooperation with the Technical University of Munich, Munich, Germany.

Since 1997, he has been with German Aerospace Center (DLR), Oberpfaffenhofen, Germany. His current areas of research include conformal antennas, smart antennas, and numerical techniques in electromagnetics.



Achim Dreher (M'92–SM'99) was born in Hermannsburg, Germany, in 1955. He received the Dipl.-Ing. (M.S.) degree from the Technische Universität, Braunschweig, Germany, in 1983, and the Dr.-Ing. (Ph.D.) degree from FernUniversität, Hagen, Gmernay, in 1992, both in electrical engineering.

From 1983 to 1985, he was a Development Engineer with Rohde & Schwarz GmbH & Compnay, KG, Munich, Germany. From 1985 to 1992, he was a Research Assistant with the Department of Electrical Engineering, FernUniversität, and from 1992 to 1997, he was a Senior Research Engineer. Since 1997, he has been with German Aerospace Center (DLR), Oberpfaffenhofen, Germany, where he is currently Head of the Antenna Research Group. His current research interests include analytical and numerical techniques for conformal antennas and microwave structures, smart antennas for satellite communications and navigation, and antenna technology for radar applications.